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# CLOSED 2-FORMS AND AN EMBEDDING THEOREM FOR SYMPLECTIC MANIFOLDS

DAVID TISCHLER

The existence of universal connections was shown by Narasimhan and Ramanan [5], and Kostant [3] showed that any integral closed 2-form is the curvature form of a connection on some circle bundle. These results can be combined to show the existence of a universal closed 2-form with integral periods. In this paper we will use the symplectic structure of a complex projective space to give an elementary proof of this result; the precise statement is given in Theorem A. The result of Kostant is in fact a corollary of the existence of a universal closed 2-form, as is indicated below. Another immediate corollary of Theorem A is the result of Gromov [3] that closed symplectic manifolds can be symplectically immersed in $CP^n$, for large enough $n$; see Theorem B.

First we indicate why the proof which we are going to give here is a simple and natural generalization of an elementary fact about exact 2-forms. Consider the standard symplectic form $\Omega = \sum_{i=1}^n dx_i dy_i$ on $R^{2n}$. Any exact 2-form on a manifold $M$ can be induced from $\Omega$ by a mapping to $R^{2n}$ for some $n$, since any exact 2-form on $M$ can be written in the form $\sum_{i=1}^k df_i \wedge dg_i$, where $f_i$, $g_i$ are real valued functions on $M$. $CP^n$ has a symplectic structure $\Omega_0$ which is locally given by $\Omega_0 = \sum_{i=1}^n dx_i \wedge dy_i$. Furthermore, $CP^n$ is the $2n$-skeleton of an Eilenberg-MacLane space of type $K(Z, 2)$. It is thus natural to expect that any closed 2-form with integral periods can be induced from $\Omega_0$ by a map to $CP^n$, because there is some map to $CP^n$, for large $n$, which pulls back $\Omega_0$ to within an exact 2-form of the given closed 2-form. The only complication that is met in $CP^n$ to adjusting the map to account for the exact 2-form is that, unlike in $R^{2n}$, the symplectic charts on $CP^n$ have finite radius, so the $f_i, g_i$'s utilized would have to be bounded. The proof we give of Theorem A depends only on estimating the bounds on $f_i, g_i$ as $n$ becomes large.

A closed $k$-form on a manifold $M$ will be said to be integral if its de Rham cohomology class is in the image of the canonical coefficient map $H^k(M; Z) \to H^k(M; R)$.

Complex projective space $CP^n$ has a Kählerian structure, and we will denote its Kähler form by $\Omega_0^n$. The 2-form $\Omega_0^n$ can be chosen to represent a generator in the image of $H^2(CP^n; Z) \to H^2(CP^n; R)$, and we can assume that $i^*(\Omega_0^{n+k}) = \Omega_0^n$ where $i$ is the standard inclusion of $CP^n$ in $CP^{n+k}$.

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**Theorem A.** *Let $M$ be a closed manifold, and $\Omega$ an integral closed 2-form on $M$. Then there exists a map $f: M \to CP^n$, for $n$ sufficiently large, such that $f^*(\Omega_0^n) = \Omega$.*

Since $\Omega_0^n$ is the curvature form of a connection on the canonical $S^1$ bundle over $CP^n$, a map to $CP^n$ which induces a closed 2-form also induces an $S^1$ bundle. Hence we obtain

**Theorem** (*Kostant* [3]). *Every integral closed 2-form is the curvature form of a connection on an $S^1$ bundle.*

**Definition.** Let $(M, \Omega')$ and $(N, \Omega)$ denote two manifolds $M, N$ with symplectic forms $\Omega', \Omega$ respectively. A map $f: M \to N$ will be called a symplectic map from $(M, \Omega')$ to $(N, \Omega)$ if $f^*(\Omega) = \Omega'$.

**Definition.** Given a manifold $M$ and a symplectic structure $(N, \Omega)$, a map $f: M \to N$ such that $f^*(\Omega)$ is a symplectic form on $M$ will be said to be transverse to the symplectic form $\Omega$.

Any submanifold $M$ of $CP^n$ such that the inclusion $i: M \to CP^n$ is transverse to $\Omega_0^n$ will support a symplectic structure, namely $i^*(\Omega_0^n)$, which is an integral closed 2-form. The converse is also true and resembles Kodaira's embedding theorem, but with Kählerian weakened to symplectic.

Suppose $(M, \Omega)$ is a symplectic structure. If $\Omega$ is an integral closed 2-form, then by Theorem A there is a map $f: M \to CP^n$ such that $f^*(\Omega_0^n) = \Omega$. Since $\Omega$ is a nondegenerate 2-forms $f$ is automatically an immersion. This yields the result:

**Theorem B** (*Gromov* [2]). *If $\Omega$ is a symplectic structure on $M$, and $\Omega$ is an integral closed 2-form, then there exists a symplectic immersion of $M$ into $CP^n$ for sufficiently large $n$.*

**Remark.** This result can be improved to yield symplectic embeddings in the following way. Assume $n$ is large enough so that the immersions can be approximated arbitrarily closely by embeddings. Choose an embedding $g: M \to CP^n$ so that $g^*(\Omega_0^n)$ is close to $\Omega$. By Moser's theorem on the stability of symplectic forms [4], we conclude that there is a diffeomorphism $F$ of $M$ to itself such that $F^*(g^*(\Omega_0^n)) = \Omega$. Hence $g \circ F: M \to CP^n$ is the required symplectic embedding.

**Corollary.** *Given a symplectic structure $(M, \Omega)$, there is, for large enough $n$, an embedding $f; M \to CP^n$ transverse to $\Omega_0^n$, such that $f^*(\Omega_0^n)$ can be made arbitrarily close to $\Omega$ in the following sense: given a norm $\| \ \|$ on closed 2-forms and an $\varepsilon > 0$, there are a real number $k$ and an embedding $f$ such that $\|k \cdot f^*(\Omega_0^n) - \Omega\| < \varepsilon$.*

*Proof.* Choose a collection of integral closed 2-forms $\alpha_i$, $1 \le i \le d$, which define a basis for $H^2(M; R)$. Any symplectic form $\Omega$ can be written as $\Omega = \sum_{i=1}^d r_i\alpha_i + d\omega$ for some 1-form $\omega$ and real numbers $r_i$. Choose rational numbers $q_i$ such that $\Omega' = \sum_{i=1}^d q_i\alpha_i + d\omega$ satisfies $\|\Omega - \Omega'\| < \varepsilon$. There is an integer $D$ such that $D\Omega'$ is an integral 2-form. By Theorem B, $D\Omega' = f^*(\Omega_0^n)$ for some embedding $f: M \to CP^n$. The corollary follows by setting $k = 1/D$.

Before beginning the proof of Theorem A, we need to establish several notations. $C^n$ will denote $n$-dimensional complex space, $\langle\ ,\ \rangle$ the usual Hermitian inner product on $C^n$, and $|\ \ |$ the corresponding norm.

We will consider $CP^n$ as the complex lines in $C^{n+1}$ passing through the origin, and also as the quotient space of the unit sphere $S^{2n+1}$ in $C^{n+1}$ by the action of the complex numbers of norm equal to 1.

Given two points $p_1, p_2$ in $CP^n$ we denote by $\alpha(p_1, p_2)$ the angle between them viewed as real two-dimensional planes in $C^{n+1}$, ($\cos\alpha = |\langle p_1, p_2\rangle|/(|p_1|\cdot|p_2|)$ where we are now considering $p_1, p_2$ as points in $C^{n+1}$).

For each $p$ in $CP^n$, we make a choice of $x$ in $S^{2n+1}$ which represents $p$. Where it creates no confusion we will speak of $x$ in $CP^n$, and where necessary we will denote the class of $x$ in $CP^n$ by $[x]$.

For each $p$ in $CP^n$ the above choice of $x$ allows us to choose a complex hyperplane $T_x$ in $C^{n+1}$ which passes through $x$ and is orthogonal to $x$ with respect to the Hermitian metric. $T_x$ can be identified with the tangent space to $CP^n$ at $[x]$. Let $D_x$ be the subset of $CP^n$ consisting of those complex lines in $C^{n+1}$ which intersect $T_x$. The mapping from $D_x$ to $T_x$ given by sending a point in $D_x$ to its point of intersection with $T_x$ will be denoted by $S(x)$. For $\varepsilon > 0$, $T_x(\varepsilon)$ will denote all points $y$ in $T_x$ such that $|y - x| < \varepsilon$, and $S^{-1}(x)(T_x(\varepsilon))$ will be denoted by $V(x, \varepsilon)$.

Let $z = (z_0, \cdots, z_n)$ be complex coordinates on $C^{n+1}$. We can think of $C^n$ as all points $z$ in $C^{n+1}$ with $z_0 = 1$. Let $B^n(r)$ denote all points $(z_1, \cdots, z_n)$ in $C^n$ such that $\sum_{i=1}^n z_i\bar{z}_i < r^2$.

One can identify $T_x$ with $C^n$ by choosing some unitary transformation of $C^{n+1}$ which sends $x$ to $(1, 0, \cdots, 0)$ in $C^{n+1}$. Composing this map with the mapping $(z_1, \cdots, z_n) \to (1 + \sum_{i=1}^n z_i\bar{z}_i)^{-1/2}\cdot(z_1, \cdots, z_n)$ yields a diffeomorphism $H\colon T_x \to B^n(1)$. Consider the closed 2-form $\sum_{i=1}^n dx_i \wedge dy_i$ on $B^n(1)$ where $z_i = x_i + \sqrt{-1}y_i$. One can show that the Kähler form $\Omega_0^n$ on $D_x$ satisfies $\Omega_0^n = S^*(x)\circ H^*(x)(\pi^{-1}\sum_{i=1}^n dx_i \wedge dy_i)$, by using the fact that $\Omega_0^n = (i/2\pi)\partial\bar{\partial}\log(1 + \sum_{i=1}^n z_i\bar{z}_i)$ on the hyperplane $z_0 = 1$ viewed as a holomorphic cross-section of the canonical line bundle over $CP^n$; see Chern [1] for details of the Kähler structures of $CP^n$. One can think of $H(x)\circ S(x)\colon D_x \to B^n(1)$ as a symplectic chart for $CP^n$.

There is a natural inclusion $\bar{i}\colon CP^n \to CP^{n+1}$ given by the inclusion $i\colon C^{n+1} \to C^{n+2}$ defined by identifying $C^{n+1}$ as the first $n+1$ coordinates of $C^{n+2}$. The choices made above can be made compatible with the inclusion of $CP^n$ in $CP^{n+1}$ in the following sense. For a point $[x]$ in $CP^n$ we can choose $T_x, D_x$, $S(x), H(x)$ as above. We can also let $i(x) \in C^{n+2}$ represent $\bar{i}[x]$, and we have $T_x = T_{i(x)} \cap C^{n+1}$, and $S(i(x))\circ\bar{i} = i\circ S(x)\colon D_x \to T_{i(x)}$. One can also choose $H(i(x))$ so that $H(i(x))\circ i = i\circ H(x)\colon T_x \to B^{n+1}(1)$. With these choices,

$$\frac{1}{\pi}\sum_{i=1}^{n+1} dx_i \wedge dy_i = ((H(i(x))\circ S(i(x)))^{-1})^*(\Omega_0^{n+1})$$

on $B^{n+1}$, and also

$$\frac{1}{\pi}\sum_{i=1}^{n} dx_i \wedge dy_i = \pi_1^*((H(x)\circ S(x))^{-1})^*(\Omega_0^n) ,$$

where $\pi_1$ is the projection of $B^{n+1}(1)$ onto $B^n(1)$ defined by the projection of $C^{n+1}$ onto the first $n$ coordinates.

*Proof of Theorem A.* The function $f$ will be constructed in stages; the $j$th stage will be denoted $f_j$, where $0 \leq j \leq p$ for some $p$ to be chosen later. Choose $f_0: M \to CP^n$ for $n$ sufficiently large, so that $f_0^*(\Omega_0^n)$ and $\Omega$ are cohomologous. This can be done since $CP^n$ can be taken to be the $2n$-skeleton of an Eilenberg-MacLane space of type $K(Z, 2)$. Hence $\Omega = f_0^*(\Omega_0^n) + d\omega$ for some 1-form $\omega$ on $M$.

We need a couple of lemmas before we can construct the $f_j$'s.

**Lemma 1.** *Given $R > \varepsilon > 0$, there exists a $\delta > 0$ such that*

$$V(x, \varepsilon, \delta) = \{y \in CP^n \mid \alpha(y, x') < \delta \text{ for some } x' \in V(x, \varepsilon)\} \subset S^{-1}(x)(T_x(R)) .$$

*Furthermore, $\delta$ can be chosen independently of $n$.*

*Proof of Lemma* 1. The lemma follows easily from the facts that $T_x(\varepsilon) \subset T_x(R)$ and that, for $0 \leq \theta \leq \frac{1}{2}\pi$,

$$\{y \in D_x \mid \alpha(x, y) < \theta\} = S^{-1}\{z \in T_x \mid \cos\theta < |z|^{-1}\} .$$

From now on we fix a choice of $\varepsilon, R, \delta$ satisfying Lemma 1. We also choose a $\rho > 0$ such that $1 - \rho > \cos^2\delta$.

**Lemma 2.** *Given a 1-form $\omega$ on a closed manifold $M$, a finite open cover $\{W_i\}$ of $M$, an $R > 0$, and a $\rho$ such that $1 > \rho > 0$, there exist real valued functions $h_k, t_k$, $1 \leq k \leq p$ such that*

(1) $\sum_{k=1}^{p} dh_k \wedge dt_k = d\omega$,

(2) *each pair $(h_k, t_k)$ has support contained in some element of the cover $\{W_i\}$,*

(3) $\prod_{k=1}^{p} (1 + K^2(h_k^2 + t_k^2)) < 1/(1 - \rho)$, *where $K^2 = 1 + R^2$,*

(4) $h_k^2 + t_k^2 + R^2/(1 + R^2) < 1$.

*Proof of Lemma* 2. There exists some choice of functions $\bar{h}_k, \bar{t}_k$, $1 \leq k \leq \bar{p}$, such that $\sum_{k=1}^{\bar{p}} d\bar{h}_k \wedge d\bar{t}_k = d\omega$. This can be seen by choosing a partition of unity $\{\varphi_k\}$ subordinate to some finite coordinate cover $\{U_i\}$ of $M$. Then $d\omega = d(\sum \varphi_k\omega)$, and $d(\varphi_k\omega) = \sum_{i=1}^{m} d\bar{h}_k^i \wedge d\bar{t}_k^i$ for each $k$ and some choice of $\bar{h}_k^i$, $\bar{t}_k^i$ with support in $U_i$, where $m =$ dimension of $M$. Hence (1) can be satisfied. Now choose a partition of unity $\{\Psi_i\}$, $0 \leq i \leq c$, subordinate to $\{W_i\}$. Then

$$\sum_{k=1}^{\bar{p}} d\bar{h}_k \wedge d\bar{t}_k = \sum_{k=1}^{\bar{p}}\sum_{j=1}^{c}\sum_{i=1}^{c} d(\Psi_i h_k) \wedge d(\Psi_j t_k) ,$$

and (2) can also be satisfied by taking the $\Psi_i\bar{h}_k$ as the $h_k$'s and the $\Psi_j\bar{t}_k$ as the

$t_k$'s. By replacing $h_k$ and $t_k$ by $N$ copies of $h_k/N$ and $t_k/N$ respectively, and using the fact that $\lim_{n\to\infty}(1 + n^{-2})^n = 1$, we see that we can choose the $h_k$'s and $t_k$'s to satisfy condition (3). By a similar argument, the $h_k$'s and $t_k$'s can be chosen small enough so that condition (4) is satisfied as well, and the proof of the lemma is complete.

$M$ has an open cover given by $\{f_0^{-1}(V(x, \varepsilon))\}$, $[x] \in CP^n$. Fix a finite subcover $\{W_i\}$ of this cover. Fix a choice of $\{h_k, t_k\}$, $1 \leq k \leq p$, satisfying Lemma 2 applied to our fixed choices of $\varepsilon, R, \delta, \rho, \{W_i\}$, and such that

$$\frac{1}{\pi}\sum_{i=1}^{p}(dh_k \wedge dt_k) = d\omega \qquad \text{where } d\omega = \Omega - f_0^*(\Omega_0^n) .$$

For each $k$, $1 \leq k \leq p$, we choose a $W_k$ in the cover $\{W_i\}$, such that the support of $h_k$ and $t_k$ are contained in $W_k$. Recall that $W_k = f_0^{-1}(V(x_k, \varepsilon))$ for some $x_k \in C^{n+1}$.

For each $j$, $1 \leq j \leq p$, let us assume the two induction hypotheses:

( i ) There is a map $f_{j-1}\colon M \to CP^{n+j-1}$ such that

$$f_{j-1}^*(\Omega_0^{n+j-1}) = f_0^*(\Omega_0^n) + \frac{1}{\pi}\sum_{k=1}^{j-1}(dh_k) \wedge (dt_k) .$$

(ii) $f_i(W_j) \subset V(x_j, R)$, for all $i \leq j - 1$.

If we show that (i) is true for $f_p$, we will be done since

$$f_p^*(\Omega_0^{n+p}) = f_0^*(\Omega_0^n) + \frac{1}{\pi}\sum_{i=1}^{p}(dh_k) \wedge (dt_k) = f_0^*(\Omega_0^n) + d\omega = \Omega .$$

We already have (i) and (ii) satisfied for $j = 1$; (i) is true vacuously and (ii) follows from the fact that $V(x_j, \varepsilon) \subset V(x_j, R)$. Hence it suffices to show that given $f_{j-1}$ satisfying (i) and (ii) there is an $f_j$ satisfying (i) and (ii). Define $f_j$ as follows:

(a) On $M - W_j$, set $f_j = \bar{i} \circ f_{j-1}$ where $\bar{i}\colon CP^{n+j-1} \to CP^{n+j}$ is the inclusion.

(b) On $W_j$, we define first a map $g_j\colon W_j \to B^{n+j}(1)$ given by $\pi_1 g_j = H(x_j) \circ S(x_j) \circ f_{j-1}$ with values in $B^{n+j-1}(1)$, and by $\pi_2 g_j = h_j + \sqrt{-1}\,t_j$ with values in $B^1(1)$, where $\pi_1, \pi_2$ are the projections of $B^{n+j}(1)$ onto $B^{n+j-1}(1)$ and $B^1(1)$ respectively, induced by the projections of $C^{n+j}$ onto its first $n + j - 1$ coordinates and last coordinate respectively.

We can now define $f_j = S^{-1}(i(x_j)) \circ H^{-1}(i(x_j)) \circ g_j$, (we are taking the choices of $H(x)$, $H(i(x))$, to be compatible in the sense described just before the beginning of the proof of Theorem A).

By property (4) of Lemma 2 we have that $|(\pi_2 g_j)|^2 < (1 - R^2/(1 + R^2))$ in $B^1(1)$. By induction hypothesis (ii) applied to $f_{j-1}$ and by the fact that $H(x_j)(T_{x_j}(R)) \subset B^{n+j-1}R(1 + R^2)^{-1/2}$ we have that $|\pi_1(g_j)|^2 < R^2/(1 + R^2)$ in $B^{n+j-1}(1)$. Hence we can conclude that $g_j\colon W_j \to B^{n+j}(1)$ is well defined, and consequently that $f_j$ is well defined on $W_j$. By Lemma 2, part (2), we can conclude that $f_j$ is well defined on all of $M$. On $W_j$

$$\begin{aligned} f_j^*(\Omega_0^{n+j}) &= g_j^*((H(i(x)\circ S(I(x)))^{-1})^*(\Omega_0^{n+1}) = g_j^*\Big(\frac{1}{\pi}\sum_{i=1}^{n+j} dx_i \wedge dy_i\Big) \\ &= (\pi_1 g_j)^*\Big(\frac{1}{\pi}\sum_{i=1}^{n+j} dx_i \wedge dy_i\Big) + (\pi_2 g_j)^*\Big(\frac{1}{\pi}\sum_{i=1}^{n+j} dx_i \wedge dy_i\Big) \\ &= (H(x_j)\circ S(x_j)\circ f_{j-1})^*\Big(\frac{1}{\pi}\sum_{i=1}^{n+j-1} dx_i \wedge dy_i\Big) + \frac{1}{\pi}(dh_j \wedge dt_j) \\ &= f_{j-1}^*\Big(S^*(x_j)\circ H^*(x_j)\Big(\frac{1}{\pi}\sum_{i=1}^{n+j-1} dx_i \wedge dy_i\Big)\Big) + \frac{1}{\pi}(dh_j \wedge dt_j) \\ &= f_{j-1}^*(\Omega_0^{n+j-1}) + \frac{1}{\pi}(dh_j \wedge dt_j)\ . \end{aligned}$$

This equality follows from the compatibility conditions on $H(x_j)$ and $H(i(x_j))$ discussed just before the beginning of the proof of Theorem A. Hence we have shown that induction hypothesis (i) is satisfied for $f_j$. Therefore we will be done if we can show that $f_j(W_k) \subset V(x_k, R)$ for all $k > j$. For any $x \in W_k$ and $0 \le i \le j$, set $A_i = S(x_{i+1})(f_i(x))$ and $B_i = S(x_{i+1})(f_{i+1}(x))$. We consider the $A_i, B_i$ as all contained in $C^{n+j}$, (note that $A_i$ is a scalar multiple of $B_{i-1}$). We now add another induction hypothesis for each $j$, $1 \le j \le p$,

(iii) $\langle B_i - A_i, A_{i'}\rangle = 0$ for all $i' \le i \le j-1$.

If hypothesis (iii) is true for $j-1$, it is seen to hold for $j$, since $B_j - A_j$ is perpendicular to $C^{n+j}$ in $C^{n+j+1}$, using the construction of $f_j$ as above, and by the compatibility conditions given before the proof of Theorem A. (Hypothesis (iii) is vacuously satisfied for $f_0$.)

Given $A_i, B_i$ as above and our fixed $\rho$, we will show that $\cos^2\alpha_{j-1} > 1-\rho$, where $\alpha_i = \alpha([A_0],[B_i])$. We have

$$\cos^2\alpha_i = \left(\frac{|\langle A_0, B_i\rangle|}{|A_0|\cdot|B_i|}\right)^2 = \left(\frac{|\langle A_0, A_i\rangle|}{|A_0|\cdot|B_i|}\right)^2$$

by induction hypothesis (iii), and this expression is equal to $(\cos^2\alpha_{i-1})\,|A_i|^2/|B_i|^2$. Since $|B_i|^2 = |A_i|^2 + |B_i - A_i|^2$ and $|A_i| \ge 1$, we have that $|A_i|^2/|B_i|^2 \ge 1/(1+|B_i - A_i|^2)$. However $|B_i - A_i|^2 \le K^2(h_k{}^2 + t_k{}^2)$ with $K^2 = 1 + R^2$, by the construction of $f_{i+1}$, the definition of the map $H(x_{i+1})$, and the fact that $B_i$ and $A_i$ are in $T_{x_{i+1}}(R)$. Hence we have $\cos^2\alpha_i \ge \cos^2\alpha_{i-1}\cdot(1 + K^2(h_k{}^2 + t_k{}^2))^{-1}$, and so

$$\cos^2\alpha_{j-1} \ge \prod_{k=1}^{j-1}(1 + K^2(h_k{}^2 + t_k{}^2)^{-1})\ ,$$

which is greater than $1-\rho$ by part (3) of Lemma 2. Since we chose $\rho$ such that $1 - \rho > \cos^2 \delta$, we have $\alpha_i < \delta$. Since $A_0$ is contained in $V(x_k, \varepsilon)$, we get that $B_{j-1}$ is contained in $V(x_k, \varepsilon, \delta)$ which is contained in $V(x_k, R)$ by Lemma 1. Hence $f_j(x)$ is contained in $V(x_k, R)$ for all $x$ in $W_k$. This shows that $f_j$ satisfies induction hypothesis (ii), and the proof of Theorem A is complete.

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QUEENS COLLEGE, CITY UNIVERSITY OF NEW YORK